

Introduction

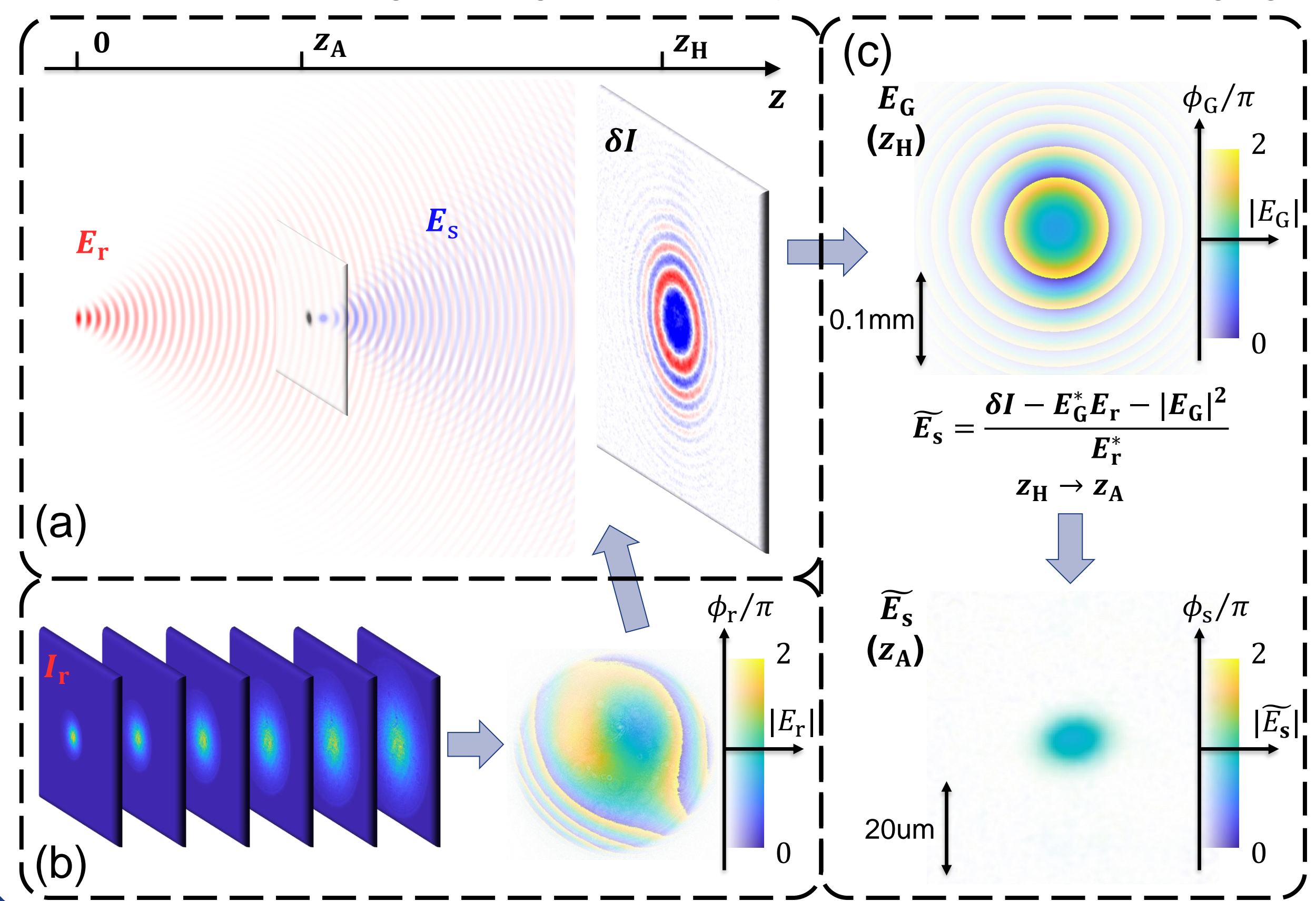
When subjected to Gaussian beam illumination, the coherent forward emission from cold atomic ensembles can usually be decomposed into a few complex Gaussian profiles. The decomposition greatly facilitates the application of a priori knowledge on the quasi-thermal atomic samples, for efficient removal of the twin-image noise during inline holography. The method supports diffraction limited complex spectroscopic imaging with shot-noise limited sensitivity. The holograms are recorded simply by defocusing the standard absorption imaging setups. We discuss the limitations of the method, and demonstrate spatially resolved atomic spectroscopy with interferometric accuracy.

Phase recovery in inline holography:

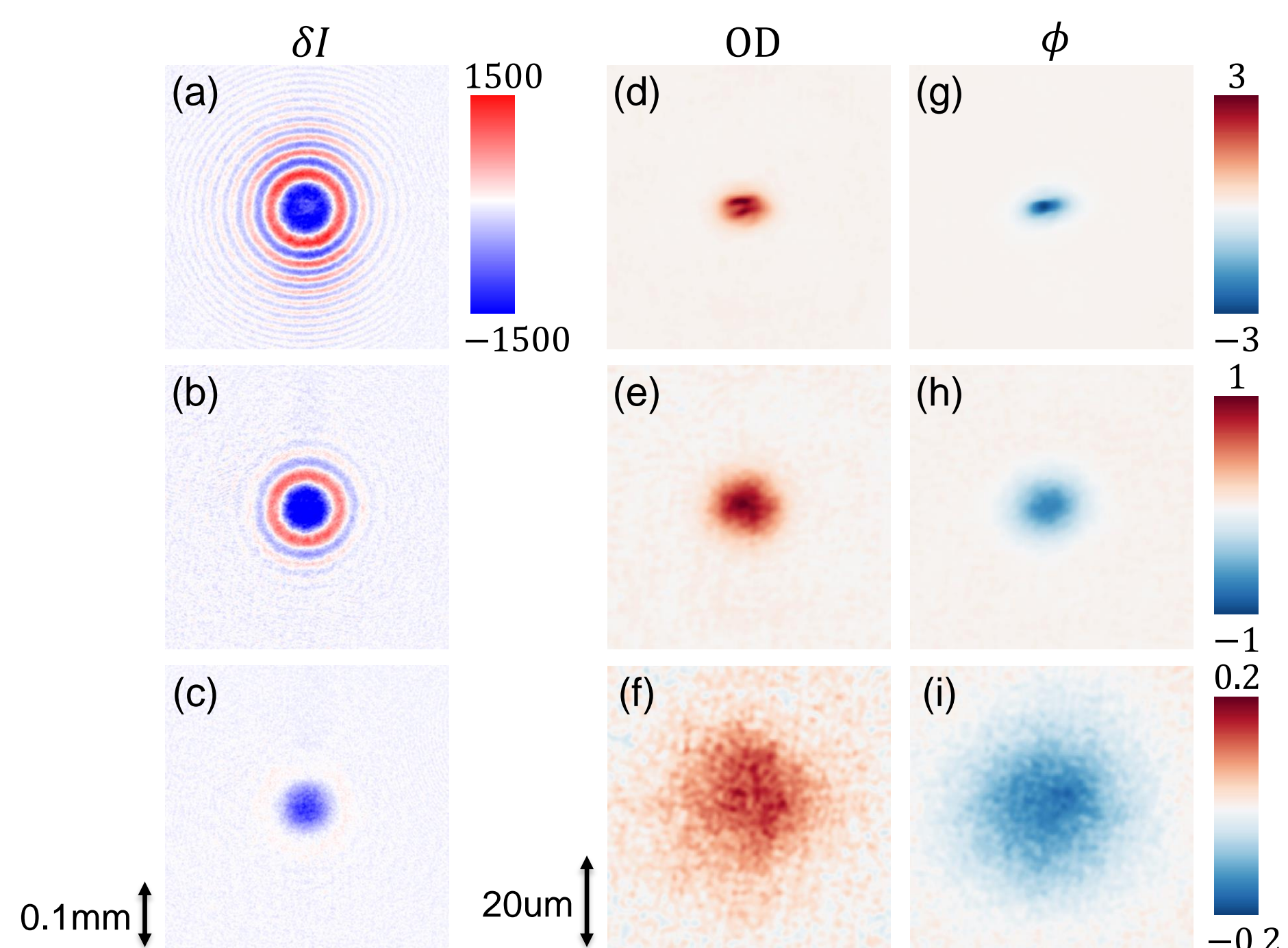
- a) **Direct:** Transportation-of-Intensity for specific samples
 - b) **Iterative:** solution under multi-plane (spatial) constraints [1,2]
 - c) **Model-fit** (compressive sensing): minimization of cost function.
- **This work:** c+b

Gaussian-packet assisted holography

1. Ref. field E_r pre-characterization (multi-plane Gerchberg–Saxton [2])
2. Gaussian packet parametrization of forward scattering: $E_s \approx E_G = \sum c_j G_j$
3. Minimization the cost function: $L = |E_r + E_G|^2 - |E_r|^2 - \delta I$.
4. Seeding the twin-removal iteration [1,2] with E_G to obtain $\tilde{E}_s \approx E_s$ (improvable with more iterations).
5. Numerical focusing: propagate E_r and \tilde{E}_s to sample plane for imaging.

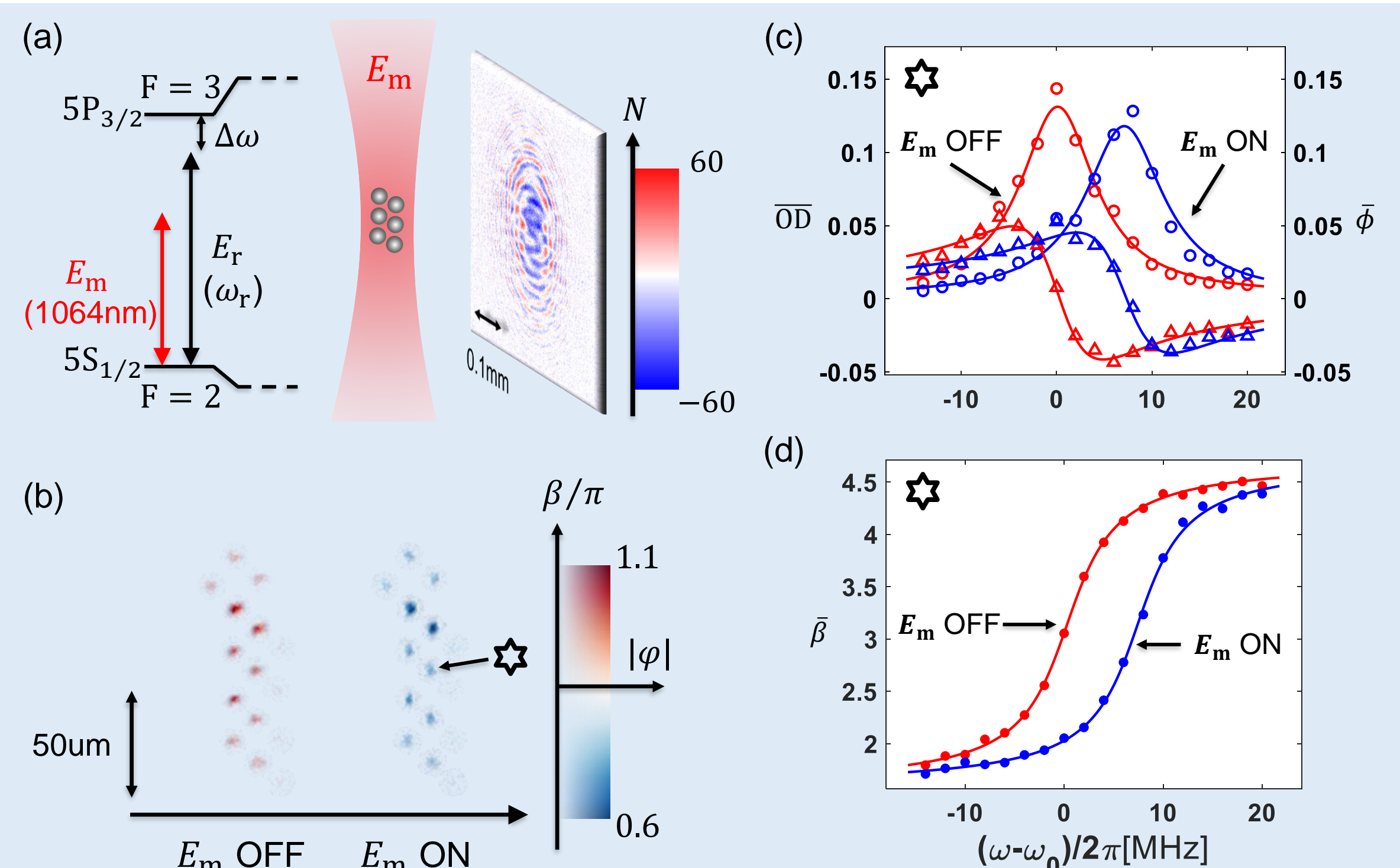


Derive OD and ϕ images from single holograms

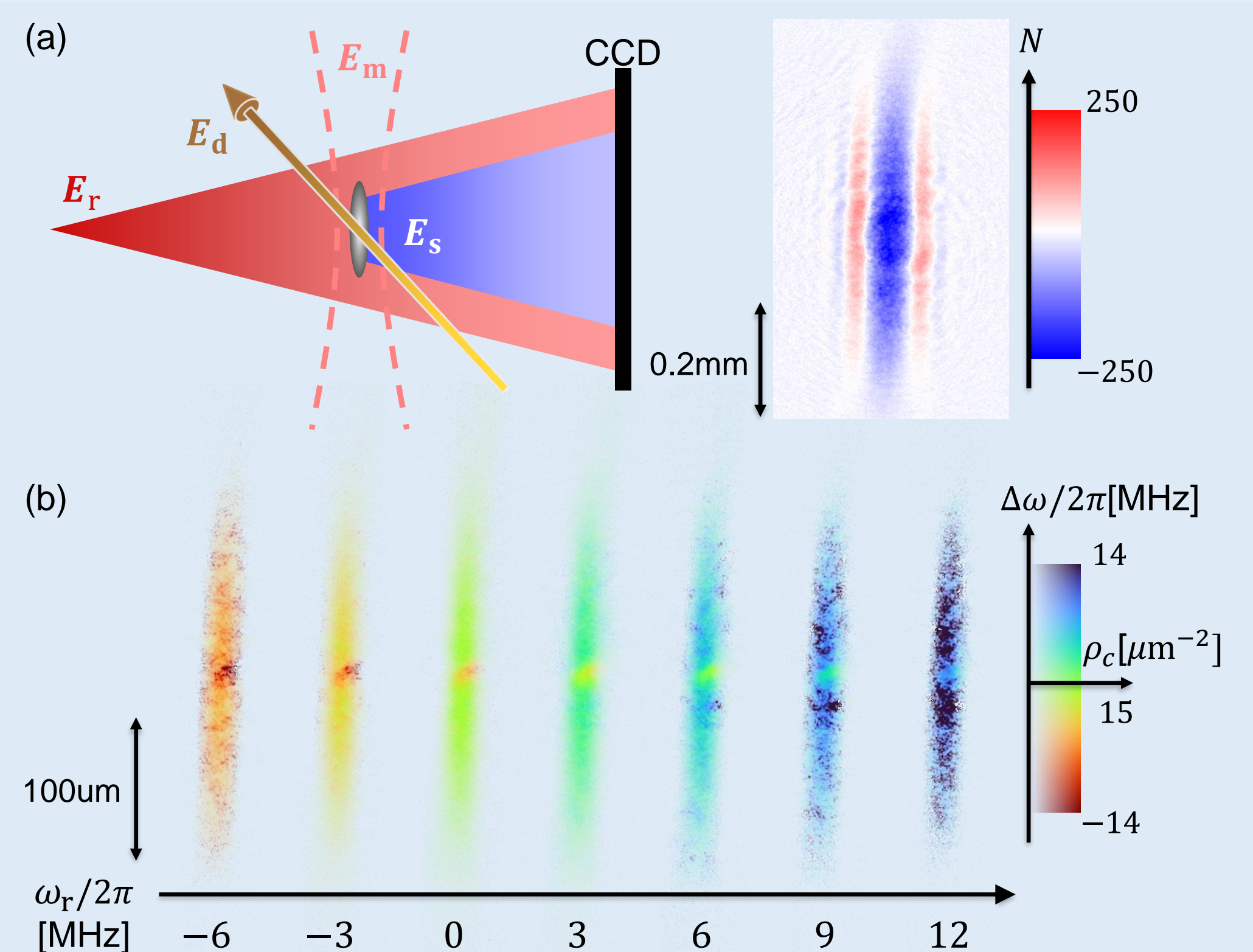


- $E_s = E_r(e^{i\phi} - 1) = E_r \sum_n \frac{(i\phi)^n}{n!} \approx E_r \sum_n c_n G^n$ (single Gaussian here)
- Optical depth $OD = 2\text{Im}[\phi]$; Phase shift $\phi = \text{Re}[\phi]$

Complex wavefront retrieval to sense field



- In this example, ^{87}Rb atomic samples are prepared in a sparse lattice, subjected to far-detuned E_m induces Stark shift (a)
- Each sub-sample in (b) is fitted with four Gaussian packets.
- Color domain plot (b) of the complex phase shift $\phi \equiv -\frac{OD}{2} + i\phi$ is retrieved from a single hologram in (a) using the Gaussian method.
- ROI-Averaged OD and ϕ spectrum (c) and phase angle $\beta = \arg(\phi)$ spectrum (d). β has a high quality immune to atom number fluctuations.



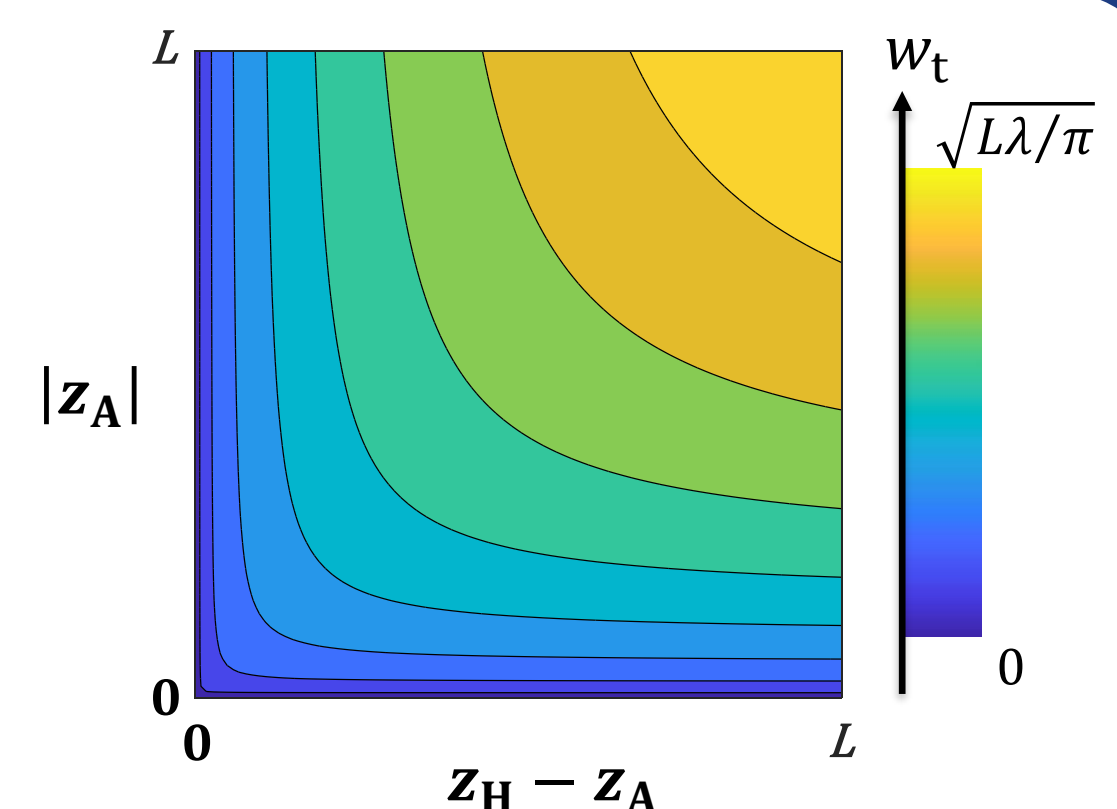
- Complex ϕ -imaging of an elongated ^{87}Rb sample dressed by a decoration beam E_d (840 nm). Here E_s is decomposed into 16 complex Gaussians: 8 to fit the overall profile and 8 for the center dressed area.

Sample size limit

The phase-recovery here relies on localized atomic sample. To avoid phase-ambiguity, it can be shown that the largest Gaussian waist for the E_s decomposition is constrained by

$$w_j < w_t = \sqrt{\frac{2\lambda|z_A|(z_H - z_A)}{\pi z_H}}$$

- Additional info, such as E_r complexity [1], can help to break this limit.



Summary and outlook

- For “smooth” atomic samples, Gaussian packet decompose is powerful for phase recovery with prior knowledge, such as shape constraints.
- In future work, aberrations by complex imaging lenses can also be accurately traced.
- The method provides a convenient path to single-shot precise sensing of atomic shifts with spectroscopic imaging, near the fundamental limits.

[1] J. P. Sobol and S. Wu, New J. Phys., 16, 093064 (2015).
 [2] Y. Wang, J. Zhao, X. Huang et al, arxiv:2105.14832.